

Jordan School District Special Program

1st Edition: January 2010

Sponsored by the Gifted/Talented Department

Acknowledgements

THANK YOU to ALL teachers in Jordan School District who motivate their students to strive for high levels of performance in rigorous problem solving experiences!



Problem Solving Philosophy

- There are many ways to solve a reasoning problem. Individual approaches should be recognized and valued.
- Thinking and reasoning are highly valued.
- Verbalizing one's thoughts, debating, and observing others are all essential to growth in problem solving. Therefore, cooperative tasks should be encouraged.
- Children need many coached and non-threatening experiences to internalize problem-solving strategies prior to the challenge of competition or evaluation.
- It is important to use words and symbols to represent the thinking process from start to finish.

Curriculum Connections

Problem solving is the cornerstone of mathematics!

Problem solving is imbedded in each grade level math core through the use of the Intended Learning Outcomes or ILO's. The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world. The ILO's for Mathematics:

- Develop a **positive learning attitude** toward mathematics.
- Become effective **problem solvers** by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
- <u>**Reason**</u> logically, using inductive and deductive strategies, and justify conclusions.
- <u>Communicate</u> mathematical ideas and arguments coherently to peers, teachers, and others using precise language and notation of mathematics.
- <u>**Connect**</u> mathematical ideas within mathematics, to other disciplines, and to everyday experiences.
- <u>**Represent**</u> mathematical ideas in a variety of ways.

"It's not that I'm so smart, it's just that I stay with problems longer." Albert Einstein

What is Monster Math?

Monster Math is an integrated program combining skills in reading, thinking, communicating, computing, conceptualizing, and problem solving. Students work in groups to solve complex story problems. An emphasis is placed on explaining and documenting reasoning and approaches to how the problem was solved.

The Monster Math program includes the following two components:

- Steps and Strategies for teaching problem solving to ALL students.
- **Competition** for identified, accelerated students. *While the problem solving process is important for all students a competition is available to students who enjoy rigor and challenge at a greater intensity.*

What is a Monster Math Problem?

Monster Math problems are OPEN-ENDED:

- A suitable course of action is not immediately apparent
- Not usually one right way to get the answer
- Not always one right answer
- Problem solver/s can choose from several viable strategies to solve the problem
- Problem solver/s usually must perform MULTIPLE STEPS to arrive to an answer

Two types of open-ended problems:

PROCESS Problem

Guess/cheo pattern, w organized	<u>S</u> ck/r ork list	evis evis bac t, etc	e <mark>t a</mark> e, dr kwa	<u>stra</u> aw a rd, s	ateg a pi olvo	<mark>gy or strategies</mark> icture, make a table, look fo e a simpler problem, make	or a an
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	5	+4	+3	+2	+1	+0 =15 games played.	

SITUATIONAL Problem

<u>Collect data outside of the problem to solve</u> Formulate a clear statement of problem and sub problems. Identify the data needed for finding a solution.

SAMPLE PROBLEM

You and a friend decide to operate a lemonade stand. You need to decide what to charge for each glass of lemonade.

Problem Solving STEPS and STRATEGIES

<u>Study</u>: Take time to study the problem

- Read the problem and state in your own words what you need to find out
 - "I need to find out _
 - "The main question I need to answer is
- Determine a label that is appropriate for the answer
 - ____trips _____feet ____pieces of candy.
- Lightly cross out any unneeded data and/or circle important information.
- List hidden sub-problems
 - Convert minutes to hours
- Determine a rough estimate of the answer
 - More than 5 trips, less than 10 feet, about 25 pieces of candy.
 - Use the estimation later to check the reasonableness of the answer.

Explore: Spend time trying strategies

- Draw a picture of the problem
- Try it with objects
- Act it out
- Work it backwards
- Chart the data (make a chart, graph, etc.)
- Check for patterns
- Try it with smaller numbers
- Guess and check

<u>Record</u>: Record and label <u>all steps</u>

- Label numbers, drawings, graph, charts
- Perform the operations and explain
- Put the answer on the answer line with an appropriate label. Be sure that all steps shown, lead to this answer.

<u>Check</u>: Look back for completion

- Read the problem again
- Check again for hidden sub-problems
- Does the answer fit the question(s)?
- Does the answer make sense?
- Is all work labeled?
- Are all steps included?

Explain thinking and reasoning throughout to be sure your work makes sense.

- See the Bulletin Board Prompts for Steps and Strategies The part judges will see in
- the Monster Math contest.

Problem Solving Performance Rubric

20 points

The answer is correct, written on the answer line, and labeled. All steps in the explanation are shown, labeled, and follow logically to the solution.

15 points

The answer is correct but...

Label or answer is missing on the answer line or Labels are missing in the work shown or A step in the process is not shown

10 points

Correct answer, but not enough work shown to support the answer/ or

Incorrect answer, but work is shown, labeled, and explained *Computation error, incorrect labels, incorrect operation, etc.*

5 points

Correct answer, but little work is shown and no explanation is given to show how students arrived to their answer /or

Incorrect answer or no answer, limited work to suggest students read the problem and attempted to solve it.

Problem Solving Achievement Scale

(Total score for 2 problems)

LevelPoEinstein40	<u>oints</u> D
Wizard 30)-35
Champ 20)-25
Star 10)-15
Participation 5	



Second Grade Problem Solving Problem Sample Responses /Points Earned

Billy has some pet birds and dogs. He has 8 pets altogether. If Billy's pets have a total of 24 legs, how many of his pets are birds? How many are dogs?



Incorrect answer or no answer, limited work suggesting student(s) read problem & attempted to solve.

A CORRECT answer on the answer line with little/no work would also earn 5 points.

Teacher Directions

The directions and following problems are ones which can be done as a whole class working together, or in small groups. Most of the questions should be read and discussed together as a class, with time given for students to generate a solution. The most important part of these lessons is to teach the process, not everyone getting the one, "right" answer.



Problem Solvers In Training Teachers Instructions

Take the time to review the question. Decide on materials needed, essential concepts and vocabulary to help students solve the problem.

MATERIALS:

Make an overhead of the attached worksheet and problem solving steps. Each pair of students working as partners needs counters, pictures, or baking size mini M & M's, a pencil, and a copy of the problem.

CONCEPTS AND STRATEGIES THIS LESSON:

Key words: Choose words that may need some additional explanation to help students understand the problem

Teach the students to:

- Approach problem solving by analyzing the information given.
- Decide what the problem is asking you to do and what the answer should look like
- Find the best strategy to solve the problem; acting it out, using counters, etc.
- Record all work done to how the answer was reached.
- Record answer in the correct space.
- Teach the students to use cooperative behavior when working with a partner:
 - 1. Share the counters.
 - 2. Listen to partners.
 - 3. Allow partner to show you how he/she would work it.

PROCEDURE:

1. Show the students the Monster Math steps of problem solving. Tell them, "We will follow these steps to solve this problem." (Later you may want to put advance students on their own, but if you do that too soon they won't follow the process and the problems are written so they need to do so.)



- 2. Chorally read the problem once. Have them turn their paper over and tell you what they remember. Ask other students to agree or disagree with what the various volunteers remember. Read the problem again discuss disagreements.
- 3. Read the problem the second time. Then ask "What are we supposed to figure out here?" Get them to disagree with you by asking something ridiculous like, "Are we supposed to find out how many farms, farmers, trucks, etc.?" Keep this up until they tell you over and over what they need to find out. This is a good time to discuss the key words students need to understand to solve the problem. What do these words suggest the students do to solve the problem? Ask them to decide what the answer will look like for the problem.

PROCEDURE:





the problem. Lead the children to identify and circle information that will help them to find the answer.5. Ask students "How could we work this out?" Suggest using objects, pictures or

will not help us solve the problem. Demonstrate, on an overhead or document camera, how to cross out the information that will not help the students solve

4. Read the problem a third time. Tell the students to look for "information " that

- Ask students "How could we work this out?" Suggest using objects, pictures of let them act out the problem. Have in mind one or two strategies that would help them solve the problem. Allow students to choose the strategy that would best help them. Also help them to see if there may be any sub-problems that may need to be solved first.



- 6. After students have worked on the problem. discuss how they could show this on paper. Explain to them it isn't necessary to make exact drawings of the items they used. Help them to come up with a representation of the manipulative used; i.e., drawing squares to represent unifix cubes.
- 7. When they have finished check it as a class, using an overhead or document camera. Check the answer by reading the problem again. Use the questions below to help students correct their work:
 - Does the answer fit the question?
 - Does the answer make sense?
 - Is all work labeled?
 - Are all the steps included?

Have them check their drawing and their counting. Now would be a good time to help them understand that their answer will probably need to be in the form of a sentence. Then remind them to put the answer on the answer line.

8. To end the lesson, ask the students what they learned working this problem. Write their responses in a class journal: "What we're learning about problem solving," along with the date and their papers. Over time the steps should become automatic, but each problem extends thinking and problem solving ability.

*As a challenge to gifted math students, pose new questions, such as increase the number of animals in the barn.

Problem Solvers In Training Practice Problems

The following problems are for you to use to practice problem-solving strategies in your classroom. Many of these questions are designed to help students practice 1 problem solving strategy. Some problems are easy others may be more challenging for your class.

You can also access more problems on the GT website:

http://t4.jordandistrict.org/gandt/index.php.



The color bears all live on the same street, next door to each other. The color bears are green, yellow, blue and red. Red bear lives between yellow bear and green bear. Green bear does not live in the first house. Yellow bear lives in the last house. In what order do the color bears live? Show and label all work.

Answer _____

Show, label, and explain **all** work:



The following problems were written or adapted by Deanna Taylor. All rights reserved.

1

If Jack and Jill each invite 2 friends over to play four square at 4:00, how many girls could be there? Show and label all work.

Answer _____

Jack and Jill have 4 buckets they can use to get water. They can each carry two buckets up the hill, but they each can carry only one down full. They need 5 buckets of water for lemonade. How many trips up and down the hill will it take Jack and Jill to get 5 full buckets of water? Show and label your work.

Answer _____

Our class has 26 desks in it. The desks are to be arranged in tables. How many different table arrangements can be made in our class? No table can have less then 2 desks or more than 5. Show and explain your work.

Answer _____

On Halloween Street a witch, ghost, vampire, monster and mummy all live next to each other. The mummy does not live in the middle house, but has a nice garden. The last house belongs to the monster. The witch lives between the ghost and the vampire. They do not get along. The ghost lives in the second house. What order are the houses these characters live in on Halloween Street? Show and explain your work.



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Wilma the Witch was making her famous Halloween Laughing Brew. She put 13 items in her brew. She put in 3 spiders, 2 frogs, 1 lizard, and the rest bats. Then Wilma was confused; she couldn't remember what she put in. She reached in the brew and pulled out a frog. Then she pulled out the lizard. What would be the most likely item for Wilma to pull out next? Show and label all work.

Answer _____

Show, label, and explain **all** work:

6

Farmer Mac has 10 turkeys on his farm. They like to hide behind the barn, under the haystack, inside the root cellar, or up in a tree. If all the turkeys are hiding, how many could hide in each area? Show and label your work. Record your answer on the answer line.

Answer _____

Jack and Jill see 5 trees in the forest with 18 pinecones on them. One tree didn't have any pinecones. How many pinecones could there be on each tree? Use the trees below to show and label your work.

Answer _____

Show, label, and explain **all** work

The following problems were written or adapted by Deanna Taylor. All rights reserved.

Santa and Mrs. Claus have 7 trees to decorate for Christmas. They have only 30 ornaments to use on all the trees. Every tree needs to be decorated for Christmas. How many ornaments could they put on each tree so they are all decorated for Christmas?

Answer ___

Santa and his elves were getting ready for Christmas. Each day they pack away the toys made. On the first day, 2 toys were made. On the second day 4 toys were made and packed away. 6 toys were made on the third day. How many toys were made on the eighth day? How many toys were made in one week? Show and explain your work.

Answer

Show, label, and explain **all** work

10

There is a hungry shark in the ocean. On Sunday he ate one fish. On Monday he ate two fish. Every day he ate one more fish than the day before. How many did he eat on Friday? Make a chart to show and label your work.

Answer _____

Susie had to deliver Valentines to her 4 friends. She made each friend's Valentine a different color. Tom's Valentine was red, Jace's was white, Jaymie's was pink, and Samantha's was purple. On Valentine's Day Susie was too sick to deliver her Valentines. Susie's mom decided to deliver the Valentines for her. Susie told her mom that Jace did not live in the first house. Tom lives between Jace and Samantha. Tom's house has roses in front. Samantha lives in the last house. Please help Susie's Mom deliver the Valentines to the right house.

Answer____

Tom began delivering his Valentines at 12:00 p.m. At 1:00 p.m. he had delivered 3 Valentines. At 2:00 p.m. he had delivered 6 Valentines. At 3:00 he had delivered 9 of his Valentines. If Tom can deliver 3 Valentines every hour, how many Valentines will he have delivered by 6:00 p.m.? Use the pictures below to show and explain your work. Be sure to Label all answers completely and carefully.

Answer _____

Peter, Jane and Jack each have a section in the garden at their house. In each section they planted some red and yellow tulips. The tulips have all started to bloom. 7 out of 10 tulips in Peter's garden are red. 3 out of 10 tulips in Jane's garden are yellow, and 6 out of 10 tulips are red in Jack's garden. How many yellow tulips does each garden have? How many yellow tulips are there all together? Show and explain all your work. Be sure to label your answer completely on the line.

Answer_

Strega Nona has a magic pasta pot that makes pasta. The pasta pot can make 6 bowls of pasta every hour. Strega Nona wants to have a pasta party for her friends. She has invited 36 people to her party. How many hours will she need to make the pasta pot work, so she will have enough pasta for each person? Each person will only eat one bowl of pasta. Show and label all work. Answer:

Strega Nona told Big Anthony he had to eat all the pasta he put on the town. The villagers put the pasta into 32 large pasta bowls. Big Anthony can eat four bowls of pasta every hour. Big Anthony starts eating at 12:00 P.M. (noon). How many hours will it take Big Anthony to eat all the bowls of pasta? Show and label all work. Answer:

Catherine Leprechaun puts into a pot of gold 2 dimes and five nickels the first hour she is working. The next hour Catherine puts in double the amount she had before of each coin. How many of each coin did she put in the pot? How much money is in the pot? Use the space below to show and explain your work. Be sure to label your answer completely.

Answer: _____

Jakob loves the snow. It snowed the entire day on Jan. 4th. It started snowing at 4:00 a.m. At 8:00 a.m. Jakob put a yardstick in the ground to measure how much snow had fallen. If it was snowing 1/2 inch every hour, how many inches of snow had fallen by 8:00 a.m.? Show and label all work. Answer:

Show, label, and explain **all** work:

18

19

Jace, Tom, Alyssa and Jaymie love to collect Pokemon cards. Jace has 20 cards, Tom has 10, Alyssa has 8 and Jaymie has 6 cards. They all want to have the same number of cards? How can they share the cards so each person has the same amount? How many cards will each person need? Show and label all work. Answer:

20

There are about 16 rooms on the main floor of the Smithsonian Museum of Natural History in Washington, D.C. If it takes 20 minutes to go through each room, how many rooms can you go into starting at 9:00 a.m. and ending at 12:00 p.m.? Answer:

Sally has a dog, cat, moose, and turkey Beanie Baby. They are arranged on a shelf in her room. The dog is not next to the cat. The turkey is not last. The moose likes to be next to the turkey. The cat is first on the shelf. In what order are they arranged on Sally's shelf? Answer:

If Rob had the digits 4, 5 and 6, what could be all the possible two and three digit numbers he could make using each digit only once?

Answer: _____

Lucky the Leprechaun planted 15 shamrocks around his house. He planted some in a pot, some in a wheelbarrow, some by his pot of gold, some by his shed and some in the garden behind his house. How many could he have planted in each area?

Answer: _____

Show, label, and explain **all** work

23

Lucy likes to make necklaces for her friends. She uses purple, green and red Fruit Loops to make the necklaces. If Lucy uses at least 2 colors for each necklace, what are the possible necklace patterns she could make? Show and label all your work.

Answer: _____

A hedgehog, badger, fox, and mouse all live in a mitten. The badger does not like to be last. The hedgehog likes to be next to the mouse. The fox likes to be first. In what order would the animals enter the mitten?

Answer: _____

On a farm there is a barn. Inside the barn there are a spider, two baby chicks, a hen, and a horse. The farmer and his son are outside in the cornfield. How many legs are there in the barn? Answer:

Corduroy decided he wanted 3 new buttons on the front of his coveralls. If Corduroy used red, blue, and yellow buttons; how many different ways could the buttons be ordered on Corduroy's coveralls? All three colors need to be used only once on each coverall. Be sure to watch for repeated patterns.

Answer: ____

Show, label, and explain **all** work



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Mrs. Pearce's class likes to use dominoes. Mrs. Pearce chose 2 dominoes, and the total number of dots on them together was 9. Which two dominoes could she have chosen? Answer: _____

On Monday Tom and Kim were putting chairs in a circle. They each carried two chairs. They each asked a friend to carry two chairs. Some of the chairs were brown and some were blue. How many blue chairs could there be in the circle?

Answer: _____

In the art room all the chairs are red and blue. There are four rows with six chairs in each row. Each row has a different pattern of colors in the way the chairs are lined up, but there are only eight blue chairs. How many chairs are red?

Answer: _____

Problem Solvers In Training Practice Problem Solutions

The following are solutions to the practice problems. Several problems will have more then one solution.



Solutions

1. The order of the color bears is: blue, green, red, yellow.

2.

Jack	b, b	b, g	b, b	b,b	g`à	b,g
Jill	g, g	b, g	b, g	b,b	g`a	g`à
#girls	3	3	2	1	5	4

b = boys g= girls *don't for get to count Jill

- 3. 6 trips 3 going up and 3 coming down
- 4. Many possible variations and combinations of desks. Look for correct number of desks, no groupings less than 2 or more than 5.
- 5. The order on Halloween Street is: mummy, ghost, witch, vampire, monster
- First find out the number of bats: 13-3-2-1= 7.
 7 bats most likely pull out a bat.
- 7. Many possible solutions. Make sure an addition problem is shown as part of solution.

Root cellar	Haystack	Barn	Tree	Total
6	2	1	1	10
5	2	2	1	10

8. Many possible solutions: make sure an addition problem is shown as part of the solution.

Tree 1	Tree 2	Tree 3	Tree 4	Tree 5	Total
5	5	2	3	3	18
2	4	4	4	4	18

9. Many possible solutions. Make sure an addition problem is shown as part of solution.

Tree 1	Tree 2	Tree 3	Tree 4	Tree 5	Tree 6	Tree 7	Total
5	5	5	5	5	2	3	30
10	10	2	2	2	2	2	30

10. (look for patterns) 16 on $8^{\rm th}$ day, 14 on $7^{\rm th}$ day $\,$ - 7 days in a week.

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
2	4	6	8	10	12	14	15

11. 6 fish were eaten on Friday.

Monday	Tuesday	Wednesday	Thursday	Friday
2 fish	2+1=3	3+1=4	4+1=5	5+1=6

- 12. The order the Valentines should be delivered: Jaymie Jace Tom Samantha
- 13. 18 delivered by 6:00

1:00	2:00	3:00	4:00	5:00	6:00
3	6	9	12	15	18
0+3=3	3+3 =6	6+3=9	9+3=12	12+3=15	15+3=18

14. 10 - 7 = 3 10 - 3 = 7 10 - 6 = 43 + 7 + 4 = 14

7 yellow in Peter's garden

- 3 yellow in Jane's garden
- 4 yellow in Jack's garden

3 + 7 + 4 = 14 14 yellow tulips all together.

15. 6 bowls every hour. Took 6 hours to make all the pasta.

l hour	2 hour	3 hour	4 hour	5 hour	6 hour
6	12	18	24	30	36
0 + 6=6	6 + 6=12	12+6=18	18+6=24	24+6=30	30+6=36

12:00	1:00	2:00	3:00	4:00	5:00	6:00	7:00
32-4 =	28-4=	24-4=	20-4=	16-4=	12-4=	8-4=	4-4=
28	24	20	16	12	8	4	0

16. He eats 4 bowls every hour. It will take him 7 hours.

17. She put in 2 dimes and 5 nickels to begin with. Doubled 4 dimes and 10 nickels

10+10+10+10+5+5+5+5+5+5+5+5+5+5=9040+50=90

18.

4:00	5:00	6:00	7:00	8:00	Total
0	½ inch	½ inch	½ inch	½ inch	2 inches

19. 20 + 10 + 8 + 6 = 44 Each person would get 11. Jace won't need any, but needs to give away 9; 20-11=9 Tom needs 1; 10-1=11 Alyssa needs 3; 8+3=11 Jaymie needs 5; 6=5=11

20. 9:00 – 12:00 is 3 hours. Each hour has 60 min. There are 3 20 minutes in 60 minutes.

l hour	20, 20, 20	3 rooms	3
2 hour	20, 20, 20	3 rooms	3+3 = 6
3 hour	20, 20, 20	3 rooms	6+3=9

9 rooms in 3 hours.

21. Beanie order on the shelf:

cat	turkey	moose	dog
cat	moose	turkey	dog

22. Possible solutions. Watch for duplicates

45	54	46	64	56	65
456	654	546	546	645	

23. Many possible solutions. Make sure an addition problem is shown as part of solution.

In a pot	Wheelbarrow	Pot of gold	Shed	Garden	total
5	5	5	5	5	15
10	2	1	1	1	15

- 24. Some possible pattern solutions. Watch for duplicates. red, green, red, green, red, green red, green, purple, red, green, purple green, purple, green, purple, green, purple red, purple, green, red, purple, green
- 25. The order in the mitten may be: fox badger hedgehog mouse fox badger mouse hedgehog
- 26. There are 18 legs in the barn. spider - 8 legs 2 chicks - 4 legs/2+2=4 hen - 2 legs horse - 4 legs 8 + 4 + 2 + 4 = 18 legs (chicks together) 8 + 2 + 2 + 2 + 4 = 18 legs (chicks separate)

27. Six ways to arrange Corduroy's buttons. Watch for duplicates

1	2	3	4	5	6
Red	Red	Blue	Blue	Yellow	Yellow
Blue	Yellow	Yellow	Red	Red	Blue
Yellow	Blue	Red	Yellow	Blue	red

28. He could have pulled out:



29. Many possible solutions. Watch for duplicate combinations.

Tom	Kim	Friend	Friend	total
Blue, blue	Blue, blue	Brown,	Brown,	4 blue
		brown	brown	chairs
Blue,	Blue, blue	Brown,	Brown,	3 blue
brown		blue	brown	chairs

30. This is one possible solution. 16 red chairs.

	-				
Red	Blue	Red	Blue	Red	Blue
Red	Red	Blue	Red	Red	Blue
Red	Red	Red	Blue	Red	Red
Blue	Red	Red	Blue	Red	Red

Monster Math

Bulletin Board Prompts can be accessed on the Gifted and Talented website. <u>http://t4.jordandistrict.org/gandt/index.php</u>.

Bulletin Board Prompts consist of:

- Problem Solving STEPS
- Problem Solving STRATEGIES

